Introduction to Stochastic PDEs
Term 1 - 2019
Course Info

Overview

This course will be an introduction to stochastic partial differential equations (SPDEs). The beginning of the class will give some background on functional analysis and infinite dimensional Gaussian measures.
We then turn to stochastic integration in infinite dimensions and close the course with some analysis of linear and semi-linear SPDE.

Logistical information

- Instructors: Giuseppe Cannizzaro (g.cannizzaro[at]imperial.ac.uk) and Ajay Chandra (a.chandra[at]imperial.ac.uk).
- Meeting time: Fridays, 2PM->4PM, with the first lecture on 18 October and the final lecture on 6 December.
- Room: Room 402 in the CDT Space

Required Background

Students should have a solid background in analysis and some familiarity with functional analysis. They should also have some background in probability and familiarity with stochastic integration with respect to Brownian motion.

Resources

Our primary resource will be Professor Martin Hairer's lecture notes which can be found here.

Course Outline

- 18 October - Lecture 1 (Giuseppe)
    - Introducing and doing some computations with the Stochastic Heat Equation (SHE) and the Parabolic Anderson Model (PAM)
    - Defining Gaussian measures on Banach Spaces
    - Fernique's Theorem

- 25 October - Lecture 2 (Giuseppe)
    - Consequences of Fernique's Theorem
        - Covariance, moment, and tail bounds
        - Equivalence of moments
    - Kolmogorov's Theorem, application to 1d SHE
    - Motivating Cameron-Martin Theory

- 1 November - Lecture 3 (Ajay)

    - Definition of the Cameron-Martin space
    - Cameron-Martin continuity theorem and consequences
    - Borell's inequality and consequences

- 8 November - Lecture 4 (Ajay)
    - Cylindrical Wiener Processes
    - Reviewing (with a sketch) the finite dimensional stochastic integral
    - Infinite dimensional stochastic integral
    - Definition of semigroups, strongly continuous semigroups, generators

- 15 November - Lecture 5 (Giuseppe)
    - Hille-Yosida Theorem
    - Adjoint, self-adjoint operators
    - Analytic semigroups (only statements no proofs), SDE example
    - Outline of interpolation spaces

- 23 November - Lecture 6 (Giuseppe)
    - Linear PDEs
        - Existence/uniqueness
        - Interplay of space vs time regularity
        - Long time behaviour

- 30 November - Lecture 7 (Ajay)
    - Finishing discussion of long-time behavior of linear SPDE
    - Convergence to equilibrium in different topologies
    - Reaction diffusion equations

- 6 December - Lecture 8 (Ajay)
    - Review and finishing discussion of reaction diffusion equations
    - Local and global existence for Phi_4^1 equation

Assessment:
In order to be assessed in this class, students will be asked to form groups of 2 or 3 and choose one of the following topics to study and then prepare an hour long presentation on that topic to be shared with the class. Upon choosing a topic the students will be given specific materials to prepare from, please speak to us if you would like guidance on which parts of the materials should be the focus of your presentation.

The topics available are the following:

- Hermite polynomials of singular Gaussian random fields and the Da Prato - Debussche argument for Phi_4^2 - see here

- A-priori bounds on reaction diffusion equations - see here
- Approximating the stochastic Burgers equation - see here
- Martingale solutions - see here
- The stochastic wave equation - see here
- The parabolic Anderson model - see here
- Small noise asymptotics - see here