



2022 Summer School in Mathematics of Random Systems

Sunday 25th - Friday 30th September Dickson Poon Building, St Hugh's College, St Margaret's Rd, Oxford, OX2 6LE

The 2022 Summer School in Mathematics of Random Systems is jointly organised by the EPSRC CDT in Mathematics of Random Systems and the Berlin-Oxford International Research Training Group (IRTG) Stochastic Analysis in Interaction. The school will feature courses on topics of current interest in Stochastic Analysis as well as presentations by students from both Oxford and Berlin.

Stochastic Quantisation

Professor Massimiliano Gubinelli



In this minicourse I will give an overview on recent results and techniques that allow the use of stochastic analysis to study certain measures on the spaces of distributions over two and three dimensional Euclidean space which are usually known as Euclidean quantum fields. The analysis of such measures is plagued by both small scale and large scale singularities. By following basic ideas of stochastic analysis one can identify suitable building blocks and reasonably simple equations which allows the construction of such measures. This program goes under the generic name of "stochastic quantisation". We will cover basic ideas of stochastic quantisation and the relation between the properties of the measures in relation to its stochastic quantisation, e.g.: existence, uniqueness, cluster properties, etc.

References

- <u>Gubinelli, M. and Hofmanova, M (2021) A PDE Construction of the Euclidean \$\Phi^4_3\$</u> <u>Quantum Field Theory, ArXiv:1810.01700</u>
- <u>Barashkov, N. and Gubinelli, M. (2020) A Variational Method for \$\Phi^4_3\$, Duke</u> Mathematical Journal, <u>169(17): 3339–3415</u>

Additional resources:

- Barashkov, N. and Gubinelli, M. (2021) On the Variational Method for Euclidean Quantum Fields in Infinite Volume, *ArXiv*:2112.05562
- Barashkov, N. and De Vecchi, F.C. (2021) Elliptic Stochastic Quantization of Sinh-Gordon <u>QFT, ArXiv:2108.12664</u>

Optimal transport theory and Wasserstein distances

Professor Jan Obloj



The aim of the minicourse is to present recent advances related to optimal transport and its applications in mathematical finance, statistics, optimization and beyond. We will discuss basics of optimal transport (OT), its duality theory and properties of the induced Wasserstein distance on the space of probability measures. I will then introduce the martingale version of the problem (MOT) and discuss the rich additional structure resulting from the martingale constraint. I will touch on numerics for both problems, including the entropic relaxation of the OT. Finally, I will discuss how Wasserstein distances can be used to develop robust data-driven approaches to modelling in mathematical finance, machine learning and beyond.

References:

- C Villani (2009), Optimal Transport: Springer:
 - o <u>Old version</u>
 - o <u>New version</u>
- Filippo Santambrogio(2015), Optimal Transport for Applied Mathematicians, Springer
 - o Old(er) version
 - Published version
- Cedric Villani(2003), <u>Topics in Optimal Transportation, AMS</u>

MOT basics:

There are several papers here and books but there is no single basic reference work I would recommend. Instead, maybe it is helpful to point to a variety of papers which showcase the relevant topics (not all will be covered though):

- <u>Beiglböck, M., Henry-Labordère, P. & Penkner, F. Model-independent bounds for option prices—a mass</u> <u>transport approach.</u> *Finance Stoch* **17**, 477–501 (2013)
- <u>Beiglböck, M., Cox, A.M.G. & Huesmann, M. Optimal transport and Skorokhod embedding. Invent. math.</u> 208, 327–400 (2017)
- <u>Guo, G. & Obłój, J. Computational methods for martingale optimal transport problems.</u> Ann. Appl. Probab.
 <u>29 (6), 3311 3347, (2019)</u>
- Oblój, J & Wiesel, J. Robust estimation of superhedging prices. Ann. Statist. 49(1) 508 530, (2021)
- Bartl, D. Drapeau, S. Oblój, J. and Wiesel, J. Sensitivity analysis of Wasserstein distributionally robust optimization problems. *Proc. R. Soc. A.* 477 20210176 (2021)

Monday 26th September

Break			
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Woodstock Road, Oxford, OX2 6GG			

Tuesday 27th September

9:00	Stochastic Quantisation	Prof. Massimiliano Gubinelli	
10:30	Coffee Break		
11:00	Extended Mean Field Control Problems with	Robert Denkert	HU Berlin
	Singular Controls		
11:20	Analysis of the Ensemble KalmanBucy Filter	Sebastian Ertel	TU Berlin
	for correlated observation noise		
11:40	Incorporating observation and delay costs in	Jonathan Tam	University of
	MDP frameworks		Oxford
12:00	Bayesian Adaptive Optimal Stochastic	Alexander Merkel	TU Berlin
	<u>Control</u>		
12:20	Lunch		
2:00	Optimal transport theory and Wasserstein di	stances Prof. Jan Ob	loj
2:00 3:30	Optimal transport theory and Wasserstein dia Tea Break	stances Prof. Jan Ob	loj
2:00 3:30 4:00	Optimal transport theory and Wasserstein dia Tea Break Macroscopic loops in a random walk loop	stances Prof. Jan Ob Alexandra Quitmann	loj WIAS Berlin
2:00 3:30 4:00	Optimal transport theory and Wasserstein dia Tea Break <u>Macroscopic loops in a random walk loop</u> <u>soup</u>	stances Prof. Jan Ob Alexandra Quitmann	loj WIAS Berlin
2:00 3:30 4:00 4:20	Optimal transport theory and Wasserstein dia Tea Break <u>Macroscopic loops in a random walk loop</u> <u>soup</u> <u>Topologies on unparameterised path space</u>	stances Prof. Jan Ob Alexandra Quitmann Will Turner	WIAS Berlin
2:00 3:30 4:00 4:20	Optimal transport theory and Wasserstein dia Tea Break Macroscopic loops in a random walk loop soup Topologies on unparameterised path space	Alexandra Quitmann Will Turner	WIAS Berlin Imperial College London
2:00 3:30 4:00 4:20 4:40	Optimal transport theory and Wasserstein dia Tea Break Macroscopic loops in a random walk loop soup Topologies on unparameterised path space Quenched functional CLT for random walks	Alexandra Quitmann Will Turner Weile Weng	VIAS Berlin WIAS Berlin Imperial College London TU Berlin
2:00 3:30 4:00 4:20 4:40	Optimal transport theory and Wasserstein dia Tea Break Macroscopic loops in a random walk loop soup	Alexandra Quitmann Will Turner Weile Weng	loj WIAS Berlin Imperial College London TU Berlin
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2:00 3:30 4:00 4:20 4:40 5:00	Optimal transport theory and Wasserstein dia Tea Break Macroscopic loops in a random walk loop soup Topologies on unparameterised path space Quenched functional CLT for random walks in degenerate doubly stochastic environment Nearest-Neighbour Resampling for Off- Policy Policy Evaluation with Applications in	Alexandra Quitmann Will Turner Weile Weng Michael Giegrich	loj WIAS Berlin Imperial College London TU Berlin University of Oxford
2:00 3:30 4:00 4:20 4:40 5:00	Optimal transport theory and Wasserstein dia Tea Break Macroscopic loops in a random walk loop soup Topologies on unparameterised path space Quenched functional CLT for random walks in degenerate doubly stochastic environment Nearest-Neighbour Resampling for Off- Policy Policy Evaluation with Applications in Market Making	Alexandra Quitmann Will Turner Weile Weng Michael Giegrich	loj WIAS Berlin Imperial College London TU Berlin University of Oxford

Wednesday 28th September

9:00	Stochastic Quantisation	Prof. Massimiliano Gubinelli	
10:30	Coffee Break		
11:00	Optimal transport theory and Wasserstein distances Prof. Jan Obloj		
12:30	Lunch		
2:00	Walk		
	Please meet near the Porter's lodge at St Hugh's College if you would like to join this 2-3		
	hour walk		
6:30	Pre-Dinner Drinks at St Hugh's College		
7:00	Dinner at St Hugh's College		

Thursday 29th September

9:00	Stochastic Quantisation	Prof. Massimiliano Gubinelli	
10:00	Coffee Break		
10:30	Optimal transport theory and Wasserstein distance	ces Prof. Jan Oblo	j
11:30	Break		
12:00	On the nature of the convergence to quasi- ergodicity	Hugo Chu	Imperial College London
12:20	Axioms for Quantum Gauge Fields	Min Chul Lee	University of Oxford
12:40	Lunch		
2:00	Increased Regularity of Minimizers in the Study of Laplace Asymptotics	Philipp Forstner	TU Berlin
2:20	Abstract Fock space methods for weak well- posedness of some critical and subcritical singular SPDEs	Lukas Gräfner	FU Berlin
2:40	Stochastic sewing and fractional stochastic calculus	Toyomu Matsuda	FU Berlin
3:00	Tea Break		·
3:20	Markovian approximations of rough volatility models	Simon Breneis	WIAS Berlin
3:40	Merton's optimal investment problem with jump signals	Laura Koerber	TU Berlin
4:00	Tea Break		
4:20	Multilevel Function Approximator	Filippo De Angelis	University of Oxford
4:40	On \$n\$-player timing games and subgame- perfection	Emanuel Rapsch	TU Berlin
5:00	Second-Order Approximation of Limit Order Books in the Single-Scale Regime	Konstantins Starovoitovs	HU Berlin
5:20	End		

Friday 30th September

9:00	Stochastic Quantisation	Prof. Massimiliano Gubinelli
10:30	Coffee Break	
10:50	Optimal transport theory and Wasserstein distances	Prof. Jan Obloj
12:20	Takeaway Lunch & End	

Abstracts - Monday 26th September

12:20 Conservative SPDE and fluctuations for interacting particle systems

Andrea Clini, University of Oxford

Ben Fehrman, University of Oxford

Interacting particle systems have found diverse applications in mathematics and several related fields, including statistical physics, population dynamics, and machine learning. The large-scale behavior of these systems is essentially deterministic and is characterized by the solution to a nonlinear diffusion equation. This PDE furnishes a correct description of the particle system up to order zero. However, the particle process does exhibit large fluctuations away from its mean, i.e. from the solution of this PDE. Such deviations, though rare, can have significant consequences---such as a concentration of energy or the appearance of a vacuum---which make them important to understand and simulate. In particular, their behavior can be analyzed in terms of large deviation principles and central limit theorems, which respectively provide a sharp understanding of the occurrence of such fluctuations and a description of the particle system which is now correct up to order one. Both these approaches involve deterministic and stochastic variants of the above PDE.

In this talk, we first discuss these probabilistic concepts in some simpler settings and then we introduce a continuum model to simulate rare events of particle systems. Namely, we consider a precise stochastic version of the aforementioned limiting PDE, whose well-posedness has been recently obtained, and we show that the small-noise fluctuations of this SPDE around the deterministic zero-noise limit --- i.e. around the initial PDE --- behave as the fluctuations of the particle system: that is, they satisfy an identical large deviation principle and an identical central limit theorem.

12:40 Stochastic Reconstruction in the Square Increment Setting

<u>Hannes Kern</u>, TU Berlin

In a recent paper, Peter Friz, Antoine Hocquet, and Khoa Lê (2022) used the stochastic sewing Lemma to define the integration of certain stochastic objects (stochastic controlled rough paths) against rough paths. This allowed them to build a hybrid theory to deal with rough stochastic differential equations.

Analyzing similar partial differential equations gives rise to families of distributions (germs) (Fx)x2Rd with stochastic properties in the spatial variable x. We introduce a stochastic reconstruction theorem which can be applied to such germs, using its square increments.

2:30 From diffusion to reaction – Statistical Inference for semi-linear SPDEs

<u>Sascha Gaudlitz</u>, HU Berlin

Markus Reiß, HU Berlin

We will deduce an estimator for the reaction intensity in semi-linear SPDEs from the Girsanov Theorem. Consistent inference is achieved by studying a small diffusivity level, which is realistic in applications. The main result is a central limit theorem for the estimation error of a parametric estimator, from which confidence intervals can be constructed. Statistical efficiency is demonstrated by establishing local asymptotic normality. Local observations allow for non-parametric estimation of a reaction intensity varying in time and space. The statistical analysis requires advanced tools from stochastic analysis like Malliavin calculus for SPDEs and the infinite-dimensional Gaussian Poincaré inequality.

Abstracts - Monday 26th September

2:50 Edwards--Wilkinson fluctuations of a nonlinear SHE with non-integrable correlations Luca Gerolla, Imperial College London

Martin Hairer and Xue-Mei Li, Imperial College London

Recent works showed that in dimensions three and higher, large scales fluctuations of the stochastic heat equation (and KPZ) converge to a Gaussian field, given by the solution of the Edwards--Wilkinson (EW) equation. The usual driving noise considered has compactly supported (integrable) covariance. We investigate the case of a nonlinear SHE where the noise has tailed (non-integrable) spatial covariance. Depending on the covariance decay, we obtain analogue Gaussian fluctuations and weak convergence to the EW solution in appropriate Hölder topologies.

3:10 Branching Interval Partition Diffusions

Matthew Buckland, University of Oxford

We construct an interval-partition-valued diffusion from a collection of excursions sampled from the excursion measure of a real-valued diffusion, and we use a spectrally positive Lévy process to order both these excursions and their start times. At any point in time, the interval partition generated is the concatenation of intervals where each excursion alive at that point contributes an interval of size given by its value. Previous work by Forman, Pal, Rizzolo and Winkel considers self-similar interval partition diffusions – and the key aim of this work is to generalise these results by dropping the self-similarity condition. The interval partition can be interpreted as an ordered collection of individuals (intervals) alive that have varying characteristics and generate new intervals during their finite lifetimes, and hence can be viewed as a class of Crump-Mode-Jagers-type processes.

4:00 Superbrownian Motion with Dormancy

<u>Dave Jacobi</u>, TU Berlin

The majority of species exhibit a behaviour called Dormancy, in which the individuals switch into a state of low metabolic activity, that protects them from harsh environmental conditions and in this way increases their chance of survival. Therefore models from mathematical population biology have to incorporate this phenomenon. We will consider how to extend the classical Superbrownian Motion to model Dormancy and present the technique of functional Laplace transforms for scaling limits of measure valued processes. In the end we will consider further research questions such as an SPDE for the process' density among others.

4:20 Wright-Fisher diffusion with continuum many seed banks

<u>Likai Jiao</u>, HU Berlin

In this talk, we consider a neutral 2-allele Wright-Fisher diffusion with a continuum of seed banks which is formulated as an infinite dimensional stochastic evolution equation. The motivation comes from the fact that this equation is equivalent to a convolution-type stochastic Volterra equation whose convolution kernel is a c.d.f. implying the dormancy time distribution. The well-posedness of the equation and the strong Markov property of the solution can be proved but the state space is not locally compact. As a remedy, we consider the weak* topology and view the seed-bank part as a measure valued process. In this way, the state space is compact and the solution is a Feller process. This generalized Wright-Fisher diffusion is established as a Markovian lift of the corresponding stochastic Volterra equation, if time permits, we will also talk about its scaling-limit interpretation and its coalescent process.

Abstracts - Monday 26th September

4:40 Certain rough super Brownian motion and its properties

Ruhong Jin, University of Oxford

We consider scaling limit of a certain branching random walk in random environment whose off-spring distribution has infinite variance on the plane. The Laplace functional of the limit super process is given by a non-linear parabolic equation involving Anderson Hamiltonian. We give the existence and uniqueness of this non-linear equation and then the existence and uniqueness in law of the limit super process. We also give a martingale characterization of above super process and show that it possesses the compact support property.

Abstracts - Tuesday 27th September

11:00 Extended Mean Field Control Problems with Singular Controls

<u>Robert Denkert</u>, HU Berlin

We consider a novel class of extended mean field control (MFC) problems with singular controls where the costs depend on the current state, control and the joint law of the state-control- process. We derive a dynamic programming principle and use this to derive a quasi-variational inequality (QVI) for the value function in the Wasserstein space. Subsequently, we prove an approximation of general singular controls with purely regular controls. Finally, we use the master equations of the approximating regular MFC problems to establish an uniqueness result for our QVI characterisation of the value function of the MFC problem with singular controls.

11:20 Analysis of the Ensemble Kalman--Bucy Filter for correlated observation noise Sebastian Ertel, TU Berlin

Wilhelm Stannat, TU Berlin

The Ensemble Kalman–Bucy filter (EnKBF) is an important tool in the field of stochastic filtering, that aims to approximate the law of a diffusion process, called the signal, conditioned on noisy observations. This is achieved by employing a system of diffusion processes interacting through their ensemble mean and covariance.

In this talk we first derive an EnKBF applicable to the correlated noise framework, that is when the evolution of the signal and the observation process are both influenced by a common noise term.

We prove the well-posedness of the EnKBF, which in the correlated case requires controlling the (pseudo)inverse of the ensemble covariance matrix. Finally, we investigate the mean-field limit, which is given by a McKean–Vlasov equation, that only satisfies local Lipschitz condition. We prove the well-posedness of the equation and a propagation of chaos result.

This talk is based on joint work with Wilhelm Stannat (TU Berlin), that can be found on arXiv:2205.14253

11:40 Incorporating observation and delay costs in MDP frameworks <u>Jonathan Tam</u>, University of Oxford

Christoph Reisinger, University of Oxford & Dirk Becherer, HU Berlin

Markov decision processes provide a mathematical framework that incorporates the effects of randomness in decision-making. A common approach is to employ dynamic programming to obtain an optimal control in feedback form. The implicit assumption is however, that the state process is always observable. In practice such an assumption may be hard to sustain due to technical complications and expensive operations. These can be considered as extra costs in the control model, to be optimised against the quality of information flow. We consider in this talk two variations to the standard setup of Markov decision processes. The first stipulates that a cost must be paid to observe the process at each time, the second involves an information delay to the user, the period of which can be reduced at a price. The key in both setups is that the information flow is controllable alongside the dynamics of the state process. Optimal strategies will then be twofold: in the choice of observation times as well as the state control values. We show for each problem that by considering an augmented state process, the value function satisfies a system of higher-dimensional quasi-variational inequalities. In the infinite horizon regime, such systems can be numerically solved via penalty methods, which avoids numerical instabilities that can arise from policy iteration. Finally, we explore some possible extensions to the mean-field regime in the information delay problem, where agents who compete against each other are split into types, each subject to a different delay period.

Abstracts - Tuesday 27th September

12:00 Bayesian Adaptive Optimal Stochastic Control Alexander Merkel, TU Berlin

Christoph Belak, TU Berlin & Samuel Cohen, University of Oxford

Available information is key in making optimal decisions. In some situations, the available information may even be directly affected by the decisions made. We consider a basic continuous-time problem under the restriction that the controls are only allowed to depend on information generated by the controlled process, leading to a learning effect of a hidden parameter. Existence of \$\varepsilon\$-optimal feedback controls is shown via a regularization of the HJB equation.

4:00 Macroscopic loops in a random walk loop soup

<u>Alexandra Quitmann</u>, WIAS Berlin

Lorenzo Taggi, Università di Roma La Sapienza

We consider a general system of interacting random loops which includes several models of interest, such as the Spin O(N) model, the double dimer model or random lattice permutations. We discuss the system in $\$ mathbb{Z}^d, d>2\$, and present some recent results about the occurrence of macroscopic loops whose length is proportional to the volume of the system as the inverse temperature is large enough.

4:20 Topologies on unparameterised path space Will Turner, Imperial College London

Thomas Cass, Imperial College London

The signature of a path is a non-commutative exponential introduced by K.T. Chen in the 1950s, and appears as a central object in the theory of rough paths developed by T. Lyons in the 1990s. For continuous paths of bounded variation, the signature may be realised as a sequence of iterated integrals, which provides a succinct summary for multimodal, irregularly sampled, time-ordered data. The terms in the signature act as an analogue to monomials for finite dimensional data: linear functionals on the signature can be used to uniformly approximate any compactly supported continuous function on unparameterised path space (Levin, Lyons, Ni 2013). Selection of a suitable topology on the space of unparameterised paths is then key to the practical use of this approximation theory. We present new results on the properties of several candidate topologies for this space. If time permits, we will relate these results to two classical models: the fixed-time solution of a controlled differential equation, and the expected signature model of Levin, Lyons, and Ni.

4:40 Quenched functional CLT for random walks in degenerate doubly stochastic environment Weile Weng, TU Berlin

Benjamin Fehrman, University of Oxford & Martin Slowik, University of Mannheim

We consider continuous time variable speed random walks in doubly stochastic environments on $\$ mathbb{Z}^d \$ \$(d\geq 2)\$ that are stationary and ergodic. We investigate the quenched FCLT for this type of random walks in a degenerate setting, i.e. instead of the technically convenient uniform ellipticity and boundedness conditions, we assume moment conditions on the symmetric part of the speed and the stream tensor that depends only on the antisymmetric part. In this short talk, I will introduce the set-up, and briefly discuss the current results and challenges.

Abstracts - Tuesday 27th September

5:00 Nearest-Neighbour Resampling for Off-Policy Policy Evaluation with Applications in Market Making

Michael Giegrich, University of Oxford

Benjamin Fehrman, & Ben Hambly, University of Oxford

In many real-world reinforcement learning problems, a lot of observational data is available while interaction with the original learning environment itself is costly (i.e. market making in limit order books (LOB)). Off-policy policy evaluation deals with the problem of estimating the value function of a target policy using data generated by a different (observed) policy. In this paper we propose a to-our-knowledge novel procedure for off-policy policy evaluation based on a nearest-neighbour resampling algorithm for deterministic policies in feedback form. Assuming continuity in the reward distribution and the state transition kernel, the procedure exploits that similar state/action pairs are associated with similar rewards and state transitions. This data-driven procedure has the advantage of being less wasteful with data compared to importance sampling if the observed policy has no or only a little mass on the deterministic target policy. Furthermore, the method does not explicitly assume a model such as in model-based off-policy policy evaluation. Understanding the algorithm as subsampling a regression estimator, we can analyse some of its properties and we conduct further numerical experiments. We apply this algorithm to the problem of market making in pro-rata LOB markets providing a novel and data-driven way of generating synthetic LOB paths from a large scale data set of real observational LOB data.

12:10 On the nature of the convergence to quasi-ergodicity

Hugo Chu, Imperial College London

Matheus de Castro, Jeroen Lamb, Martin Rasmussen, Imperial College London

Dennis Chemnitz, Maximilian Engel, FU Berlin

The study of absorbed Markov processes has recently attracted the interest of a very active community due to their application to varied problems. In particular, an important question has been the convergence of Birkhoff averages conditioned on survival to a so-called quasi-ergodic distribution. A natural problem is then to characterise the type of convergence one can obtain in this context. In this talk, we present some complimentary results of our paper "The Lyapunov spectrum for random dynamical systems" which address this problem.

12:30 Axioms for Quantum Gauge Fields

Min Chul Lee, University of Oxford

James Glimm, Stony Brook University

The purpose of this paper/talk is to extend the classical axiom scheme for quantum field theory to include most of the known examples of quantum gauge theories. The axioms are developed in both the Euclidean and Minkowski metrics for space time. We demonstrate that the Euclidean axioms imply the Minkowski (Wightman) axioms. We also prove the reconstruction theorem for Yang-Mills quantum fields as operator-valued tempered distributions with gauge transformation properties.

This talk is based on Axioms for Quantum Gauge Fields, arXiv:2112.08575.

2:00 Increased Regularity of Minimizers in the Study of Laplace Asymptotics

Philipp Forstner, TU Berlin

Motivated by Laplace asymptotics for singular stochastic PDEs we study the regularity of the minimizer of a functional constructed from the Φ 43 equation. The proof uses a boot strapping argument to obtain a spatially smooth minimizer. We plan to use this result to define a translation operator in direction of the minimizer on models in the Φ 43-regularity structure.

2:20 Abstract Fock space methods for weak well-posedness of some critical and subcritical singular SPDEs

Lukas Gräfner, FU Berlin

Nicolas Perkowski, FU Berlin

We expand the generator approach for singular SPDEs with (Gaussian) invariant measure developed in [1]. In said work the authors construct the infinitesimal generator \mathcal{L} of the equation as an operator on the L^2 -space with respect to the (a-priori formally) invariant measure. Thus, solutions to the underlying SPDE can be defined as solutions to a corresponding martingale problem and their law is uniquely described by the semigroup generated by \mathcal{L} .

In this talk, our central tool is a perturbation-type result for semigroups on abstract Fock spaces with infinitesimal generators of the form

$\mathcal{L}_0 + \mathcal{G}$,

where the perturbation G fulfills certain structural properties. The singularity of the problem is demonstrated by the fact that, in contrast to classical perturbation results, G is not even a well-defined operator and it has no relative bound in terms of \mathcal{L}_{0} .

Applying this framework to singular SPDEs with (Gaussian) invariant measure, we can reproduce known results for several equations in the subcritical regime. Moreover, we can even show well-posedness for some scaling-critical equations, if the prefactor in front of the non-linearity is small enough.

[1] Gubinelli, M., Perkowski, N. The infinitesimal generator of the stochastic Burgers equation. Probab. Theory Relat. Fields 178, 1067 1124 (2020).

2:40 Stochastic sewing and fractional stochastic calculus

Toyomu Matsuda, FU Berlin

Nicolas Perkowski, FU Berlin

Lê (20) introduced the stochastic sewing lemma. The lemma beautifully combines the sewing lemma from the rough path theory and a martingale inequality, and have quickly became a useful tool for stochastic analysts. In this talk, we discuss an extension of Lê's stochastic sewing lemma. An advantage of our extension and some applications to the fractional Brownian motion will be explained.

3:20 Markovian approximations of rough volatility models

Simon Breneis, WIAS Berlin

Christian Bayer, WIAS Berlin

We consider rough stochastic volatility models where the variance process satisfies a stochastic Volterra equation with the fractional kernel, as in the rough Bergomi and the rough Heston model. In particular, the variance process is therefore not a Markov process or semimartingale, and has quite low Hölder-regularity. In practice, simulating such rough processes thus often results in high computational cost. To remedy this, we study approximations of stochastic Volterra equations using an \$N\$-dimensional diffusion process defined as solution to a system of ordinary stochastic differential equations. We illustrate that these approximations yield very accurate implied volatility smiles for European options even for very small \$N\$. Finally, we also discuss pricing of path-dependent options.

3:40 Merton's optimal investment problem with jump signals

Laura Koerber, TU Berlin

Peter Bank, TU Berlin

This talk presents a new framework for Merton's optimal investment problem which uses the theory of Meyer σ -fields to allow for signals that possibly warn the investor about impending jumps. With strategies no longer predictable, some care has to be taken to properly define wealth dynamics through stochastic integration. By means of dynamic programming, we solve the problem explicitly for power utilities. In a case study with Gaussian jumps, we find, for instance, that an investor may prefer to disinvest even after a mildly positive signal. Our setting also allows us to investigate whether, given the chance, it is better to improve signal quality or quantity and how much extra value can be generated from either choice.

4:20 Multilevel Function Approximator

Filippo De Angelis, University of Oxford

Mike Giles & Christoph Reisinger, University of Oxford

We consider function approximations for which the synthetic training set is generated by means of expensive numerical methods and is, thus, the dominant part of the computational cost. We show that multilevel ideas can reduce the computational cost by generating most samples with low accuracy at a corresponding low cost, with relatively few high-accuracy samples at a high cost.

As an application of the multilevel approach, we consider learning the function that maps parameters of the model and of the financial product to the price of the financial product. In the simple case of one-layer neural networks and second-order accurate finite difference methods, the computational cost to achieve accuracy O(ϵ) is reduced from O(ϵ -4-dX/2) to O(ϵ -4), where dX is the dimension of the underlying pricing PDE. The analysis is supported by numerical results showing significant computational savings.

4:40 On \$n\$-player timing games and subgame-perfection <u>Emanuel Rapsch</u>, TU Berlin Christoph Belak, TU Berlin

In this talk, I will present a new and general mathematical formalism to approach stochastic timing games in continuous time. In that, we build on the existing and still growing literature both in mathematics and economics, central contributions going back to Dynkin ('67) and Fudenberg/Tirole ('85). In contrast to that literature, we formally allow for games that do not stop once some player has acted in order to render the logics of sequential rationality (including threats and commitments) more transparent. Results on the analytic description of Markovian subgame-perfect equilibria for Markovian noise will be discussed, and, if time permits, an example of mechanism design for irreversible investment in oligopolistic markets will be sketched.

5:00 Second-Order Approximation of Limit Order Books in the Single-Scale Regime

Konstantins Starovoitovs, HU Berlin

Ulrich Horst and Dörte Kreher, HU Berlin

We consider a fully state-dependent Markovian limit order book model and derive its second-order approximation in the single-scale regime. We start with the first-order approximation and study the critical case in which the rescaling rates for time, tick size and volume increments are equal. We obtain non-degenerate first-order and second-order approximations which are driven by fluctuations both from changes in price and placement/cancellation of limit orders. The distribution-valued second-order approximation process preserves in the limit both spatial shift coming from the price fluctuations and the infinite-dimensional driver resulting from the fluctuations of the volume density functions.

