

Foundations of Stochastic Analysis

8 hour course

Overview

The course surveys the basics of stochastic calculus as a preparation for developments in the Michaelmas term courses

Prerequisites

It will be assumed that students have a good understanding of probability and measure and at least a first course in stochastic processes

Syllabus

1. Continuous martingales and Brownian motion

Basic theorems and properties of continuous martingales and continuous local martingales. Properties of Brownian motion.

2. Stochastic integration

Construction of the stochastic integral, L^2 theory and extension to local martingales and semi-martingales. Ito's formula, all in the setting of continuous martingales.

3. Useful theorems

Dambanis-Dubins-Schwarz, Girsanov, Martingale Representation

4: SDEs classical existence and uniqueness results

Strong and weak solutions. Pathwise uniqueness and uniqueness in law. Existence and uniqueness of strong solutions via picard

5. Martingale problems

Connections between SDEs and generators; weak solutions to SDEs through martingale problems

References:

There are many books on these topics:

1. Revuz and Yor - Continuous martingales and Brownian motion
2. Rogers and Williams - Diffusions, Markov processes and martingales