# Function Spaces and Distribution Theory

## Course Format

Foundation Module – Professor Gui-Qiang G. Chen

## Duration

8 hours

## Overview

This course will be an introduction, in the spirit of a user's guide, to modern techniques in analysis, which are central to the theoretical and numerical treatment of random systems.

## Learning Outcomes

Students will learn basic techniques and results about Lebesgue and Sobolev spaces, distributions and weak derivatives, embedding and trace theorems, and weak convergence.

## Synopsis

- Revision of relevant definitions and statements from functional analysis: completeness, separability, compactness and duality.
- Revision of relevant definitions and statements from Lebesgue integration theory: convergence theorems, completeness, separability and duality.
- Weak and weak* convergence in Lebesgue spaces: oscillation and concentration. Equi-integrability and Vitali's Convergence Theorem. Examples. A bounded sequence in the dual of a separable Banach space has a weak* convergent subsequence. Statement of Mazur's Lemma.
- Mollifiers and the density of smooth functions in $L^p$ for $1 \leq p < \infty$.
- Vitali's covering lemma and maximal inequalities. Lebesgue points and precise representatives.
- Distributions and distributional derivatives. Positive distributions are measures and statements of the Riesz representation theorem.
- Sobolev spaces: mollifications and weak derivatives, separability and completeness. Poincaré and Sobolev inequalities. Embedding theorems and Rellich-Kondrachov-Sobolev theorems on compactness (sketches of proofs only).
- Traces of functions with weak derivatives.

## Prerequisites

Basic Functional Analysis and Lebesgue Integration.

## Core Reading

H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Universitext, Springer, New York, 2011

L.C. Evans and R.F. Gariepy, Measure Theory and Fine Properties of Functions, CRC Press, Boca Raton, FL, 1992

E.H. Lieb and M. Loss, Analysis, 2nd Edition, Graduate Studies in Mathematics, American Mathematical Society, 2001

E.M. Stein and R. Shakarchi, Real Analysis. Measure Theory, Integration and Hilbert Spaces, Princeton Lectures in Analysis, III. Princeton University Press, Princeton, NJ, 2005