Advanced Topics in Stochastic Analysis

CDT in Mathematics of Random Systems

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This course will focus on:

1. the theory of stochastic integration for processes with jumps;
2. weak and u.c.p. convergence theory for stochastic integrals;
3. a selection of additional topics from the list: weak solutions of (non-Markovian) SDEs and martingale problems, the representation property for local martingales and BSDEs, applications to (McKean-Vlasov) optimal control.

Description

In the first part of the course, we will study the theory of stochastic integration for processes with jumps, as presented in Protter [2]. The main novelty is that we will define a semimartingale as a stochastic process which is a *good integrator* on an elementary class of processes, rather than as a process that can be written as the sum of a local martingale and an adapted process with paths of finite variation on compacts. Given the familiarity of the audience to key ideas in the foundations course, we will move at a quick pace in certain places; however, the main results will be carefully proved and motivated.

In the second part of the course, following the presentation in Jacod and Shiryaev [1], we will review the classical theory of Skorokhod topologies on the space of càdlàg processes. Then, we will have a detailed look at weak convergence for probability measures on this space (including an introduction to Wasserstein distances). Based on this, we proceed to develop a weak convergence theory for stochastic integrals with jumps.

In the final part of the course, and if time permits, we will discuss several applications/extensions of the topics covered in the previous parts of the course. These include weak solutions of non-Markovian SDEs via martingale problems, as presented in Stroock and Varadhan [3], backward stochastic differential equations, as in Zhang [4], and their applications to (McKean–Vlasov) optimal control.

Lecture notes

The course will benefit from a tailored set of lecture notes which have been kindly provided by Andreas Søjmark. Given the time constraints, the classes will only cover the most important ideas from these notes, so students are expected to familiarise themselves with parts of the notes not covered directly in class.

References

[1] J. Jacod and A.N. Shiryaev. *Limit theorems for stochastic processes*, volume 288 of *Grundlehren der mathematischen Wissenschaften*. Springer–Verlag Berlin Heidelberg, 2003.

[2] P.E. Protter. *Stochastic integration and differential equations*, volume 21 of *Stochastic modelling and applied probability*. Springer–Verlag Berlin Heidelberg, 2nd edition, 2005.

[3] D.W. Stroock and S.R.S. Varadhan. *Multidimensional diffusion processes*, volume 233 of *Grundlehren der mathematischen Wissenschaften*. Springer–Verlag Berlin Heidelberg, 1997.

[4] J. Zhang. *Backward stochastic differential equations—from linear to fully nonlinear theory*, volume 86 of *Probability theory and stochastic modelling*. Springer–Verlag New York, 2017.